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ELF PROPAGATION IN A NON-STRATIFIED EARTH-IONOSPHERE WAVEGUIDE

April 1978

E. C. Field

Final Technical Report
Contract N00014-76-C-0584

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This report analyzes extremely low-frequency (ELF) propagation under conditions where the properties of the earth-ionosphere waveguide change markedly over transverse distances comparable with the width of a Fresnel zone. An integral equation formulation is presented that can be used to obtain numerical results for most types of daytime ionospheric disturbances. Approximate solutions are given for ionospheric disturbances of the type that would occur in single-burst nuclear environments. These fullwave results

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are compared with results calculated from the widely used two-dimensional WKB approximation, which neglects transverse ionospheric gradients. It is shown that this WKB approximation gives good results for burst-heights above about 100 km, but that fullwave theory that accounts for transverse gradients must be used for lower burst altitudes. For these lower burst-heights, the WKB method seriously overstates the propagation anomaly caused by an on-path burst and understates the anomaly caused by an off-path burst.

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PREFACE

This report is part of Pacific-Sierra Research Corporation's continuing analysis of long-wave propagation in nuclear and naturally disturbed environments. It establishes the limitations of the two-dimensional WKB approximation, which is widely used to predict extremely low-frequency (ELF) system performance in nuclear environments. Development of numerical methods for solving the formalism developed herein will be addressed in subsequent phases of this analysis.

SUMMARY

This report analyzes extremely low-frequency (ELF) propagation under conditions where the properties of the earth-ionosphere waveguide change markedly over transverse distances comparable with the width of a Fresnel zone. An integral equation formulation is presented that can be used to obtain numerical results for most types of daytime ionospheric disturbances. Approximate solutions are given for ionospheric disturbances of the type that would occur in single-burst nuclear environments. These fullwave results are compared with results calculated from the widely used two-dimensional WKB approximation, which neglects transverse ionospheric gradients. It is shown that this WKB approximation gives good results for burst-heights above about 100 km, but that fullwave theory that accounts for transverse gradients must be used for lower burst altitudes. For these lower burst-heights, the WKB method seriously overstates the propagation anomaly caused by an on-path burst and understates the anomaly caused by an off-path burst.

I. INTRODUCTION

Most theoretical treatments of long-wave propagation in the earth-ionosphere waveguide ignore variations of the waveguide properties in directions transverse to the direct path between source and receiver. Even for nonstratified ionospheric conditions, the usual procedure is to account for variations along the direct path but to neglect those in the transverse directions. This procedure--henceforth referred to as the "two-dimensional WKB" approach--represents the ionosphere solely in terms of its properties along the direct path.

Of course, the above procedure is approximate and tends to overemphasize the importance of the ionospheric properties in the vicinity of the path and to underemphasize the importance of off-path ionospheric properties. For a single-burst nuclear environment, the two-dimensional WKB approach would overstate the signal degradation due to an on-path burst and understate the degradation due to an off-path burst.

The above comments notwithstanding, the two-dimensional WKB approximation usually gives good results at conventional radio frequencies because many ionospheric irregularities have lateral scale lengths at least comparable with the wavelength and first Fresnel-zone-width of conventional radio paths. Even very low-frequency (VLF) signals have wavelengths of only tens of kilometers and Fresnel-zone-widths of no more than a few hundred kilometers. Wait (*1970*) has considered the effects of a laterally localized ionospheric depression on VLF propagation.

At ELF, on the other hand, both the wavelength and the width of the first Fresnel zone can be several megameters. Many types of ionospheric irregularities exhibit considerable lateral variations over such distances and should be analyzed by fullwave methods. An example of interest here

is the ionospheric disturbance produced by a single nuclear burst detonated at an altitude of 100 km or less. Other examples include polar-cap absorption (PCA) events and the day-night terminator.

Accordingly, this report considers ELF propagation for the situation where the properties of the earth-ionospheric waveguide change markedly over lateral distances comparable with a Fresnel zone or wavelength. Section II presents an integral-equation formulation (similar to that widely used for ground-wave propagation over irregular terrain) that can be used to obtain numerical results for most types of daytime ionospheric irregularities. Section III gives an approximate solution for the special condition of a fairly weak ionospheric depression exhibiting a single peak. The approximate form used is applicable to a single-burst nuclear environment. Section IV compares the results of fullwave and WKB results for frequencies of 45 Hz and 75 Hz. Section V summarizes the conclusions and gives guidelines for when the WKB method can--and cannot--be safely used.

II. INTEGRAL EQUATION FOR ATTENUATION FUNCTION

To avoid mathematical complexities unrelated to lateral ionospheric gradients, earth curvature and the geomagnetic field are neglected. The first of these approximations is well justified at ELF, whereas the second is reasonably accurate for ambient daytime conditions and is very accurate for disturbed conditions where ionospheric reflection heights are depressed below ambient levels. The equations derived below do not, however, give an accurate description of propagation under normal nighttime conditions. A time dependence $e^{i\omega t}$ is assumed.

The fields can be derived from a vector potential, $\underline{A}$. Since ELF waves are typically radiated from a horizontal electric dipole, $\underline{A}$ has both horizontal and vertical components even when the ionosphere and earth are stratified. At ELF, however, all modes except the TEM mode are below the cutoff of the earth-ionosphere waveguide and are too heavily attenuated to propagate to useful distances. For a stratified medium, the TEM mode can be described in terms of a vector potential having only a vertical component, A_z, which can be found by solving a scalar wave equation. In that situation, one can write

$$A_z(x,y,z) = \psi_o(x,y)\, F_o(z) \quad , \tag{1}$$

where, except at the sources,

$$\left[\nabla_T^2 + k^2\, S_o^2\right] \psi_o(x,y) = 0 \tag{2}$$

and

$$\frac{d}{dz}\left[\frac{1}{n^2(z)}\,\frac{dF_o}{dz}\right] + k^2\left[1 - \frac{S_o^2}{n^2(z)}\right] F_o = 0 \quad . \tag{3}$$

In Eq. (2), ∇_T^2 is the transverse Laplacian, k is the free-space wave number, and $n^2(z)$ is the ionospheric refractive-index-profile. The eigenvalue S_o is a constant that governs the propagation and is determined by solving Eq. (3), subject to the appropriate boundary conditions. Once S_o has been determined, the lateral dependence of A_z (and essentially the fields) is given by

$$\psi_o \sim \cos\alpha \, H_1^{(2)}(kS_o r) \quad , \tag{4}$$

where r is the distance from the transmitter to the observation point, H is the Hankel function, and the assumed horizontal electric dipole is oriented at $\alpha = 0$.

In the presence of lateral ionospheric gradients, a rigorous separation of A_z in the form of Eq. (1) is not possible. Note, however, that scale lengths for vertical ionospheric variations are on the order of several kilometers, whereas those for lateral variation are usually tens-to-hundreds of kilometers or more. Thus, for many types of ionospheric disturbances, it can be argued on semiquantitative grounds that the vector potential is composed mainly of a vertical component, A_z, which is governed primarily by the local ionosphere. When this approximation is valid, the eigenvalue S is determined by solving

$$\frac{\partial}{\partial z}\left[\frac{1}{n^2(x,y,z)}\frac{\partial F}{\partial z}\right] + k^2\left[1 - \frac{S^2(x,y)}{n^2(x,y,z)}\right]F = 0 \quad , \tag{5}$$

subject to the proper boundary conditions. Unlike the eigenvalue S_o of Eq. (3), S exhibits dependence on x and y. Once $S^2(x,y)$ has been determined at a sufficient number of values of x and y, the lateral dependence, ψ, of the potential is found by solving

$$\left[\nabla_T^2 + k^2 S^2(x,y)\right] \psi(x,y) = 0 \quad , \tag{6}$$

which is obtained by analogy with Eqs. (1) through (3).

The above quasi-derivation of Eqs. (5) and (6) follows closely the treatment of Greifinger and Greifinger (*1977*), who analyzed the effects of a cylindrically symmetric disturbance on the great-circle propagation path in terms of scattering theory. At this point, we depart from the Greifinger treatment and recast the problem into the form of an integral equation. First, note that numerical methods of obtaining S from Eq. (5) for virtually any height-profile of n^2 are in abundant supply, as are numerical results for a wide variety of ambient and disturbed ionospheres (e.g., *Budden, 1961; Galejs, 1972; Pappert and Moler, 1974; Field, 1970; Wait, 1970*). We can, therefore, assume that S(x,y) is either known or readily obtainable--which allows us to concentrate on solving Eq. (6) for the lateral dependence.

The problem at hand is the calculation of the effect on propagation of an ionospheric disturbance, which can be characterized by the difference $S^2(x,y) - S_o^2$. The undisturbed quantity, ψ_o, is governed by S_o and can be assumed known through Eq. (4). Following Wait (*1964*), subtraction of Eq. (1) from Eq. (6) leads to

$$\left(\nabla_T^2 + k^2\right)\left(\psi - \psi_o\right) = k^2\left[S^2 - S_o{}^2\right]\psi \quad . \tag{7}$$

To convert Eq. (7) to the desired integral equation, we use the two-dimensional Green's function,

$$G = -i\pi\, H_0^{(2)}(kS_o|\underline{r} - \underline{r}_1|) \quad , \tag{8}$$

which satisfies

$$\left[\nabla_T^2 + k^2 S_o^2\right] G = -4\pi\ \delta(|\underline{r} - \underline{r}_1|) \quad . \tag{9}$$

In Eqs. (8) and (9), $\underline{r}$ and $\underline{r}_1$ are vectors from the origin to the observation point and to an integration point, respectively. By following the usual Green's function procedure* the following equation for ψ is obtained:

$$\psi(x,y) = \psi_o(x,y) - \frac{ik^2}{4}\iint_{-\infty}^{\infty} dx'dy'\left[S^2(x',y') - S_o^2\right] H_o^{(2)}(kS_o|\underline{r} - \underline{r}_1|)\psi(x',y') \quad . \tag{10}$$

Equation (10) is identical to an integral equation given by Wait (*1964*).

The receiver is assumed located at (x,o)--in which case, $r = x$, $r_1^2 = (x')^2 + (y')^2$, and $r_2 = |\underline{r} - \underline{r}_1| = \left[(x - x')^2 + (y')^2\right]^{\frac{1}{2}}$. It is also convenient to define a propagation function, W, which denotes the fractional amount by which ψ differs from the value, ψ_o, that it would have in the absence of an ionospheric disturbance. Specifically, the propagation function is defined by

$$\psi(x,y) = W(x,y)\ \psi_o(x,y) \quad . \tag{11}$$

Insertion of Eqs. (11) and (4) into Eq. (10) leads to the following integral equation for W:

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*This procedure consists of multiplication of Eq. (7) by G and Eq. (9) by ψ, subtraction of the resulting two equations, integration over x'y', and use of the two-dimensional Green's theorem to convert an area integral to a line integral along an infinite circle.

$$W(x,o) = 1 - \frac{ik^2}{4}\int\!\!\int_{-\infty}^{\infty} dx'dy' \left[S^2(x',y') - S_o^2\right] \cdot$$

$$\frac{H_o^{(2)}(kS_o r_2)\, H_1^{(2)}(kS_o r_1)}{H_1^{(2)}(kS_o x)} \quad \frac{x'}{\sqrt{(x')^2 + (y')^2}}\, W(x',y') \quad . \qquad (12)$$

Even for the great wavelengths at ELF, the quantities $kS_o x$, etc., are often large enough to permit use of the leading term of the asymptotic expansion to represent the Hankel functions. The resulting equation is

$$W(x,o) = 1 + \frac{k^{3/2} e^{-\pi i/4}}{2\sqrt{2\pi S_o}} \int\!\!\int_{-\infty}^{\infty} dx'dy' \left[S^2(x',y') - S_o^2\right]\left[\frac{x}{r_1 r_2}\right]^{\frac{1}{2}} \cdot$$

$$\left[\frac{x'}{r_1}\right] W(x',y') \exp\left\{-ikS_o\left[r_1 + r_2 - x\right]\right\} \quad . \qquad (13)$$

Not surprisingly, Eq. (13), which describes propagation in a laterally non-uniform earth-ionosphere waveguide, is very similar to the classic integral equation representation of ground-wave propagation over laterally non-uniform terrain (e.g., *Hufford, 1952*).

III. EQUATIONS FOR AN ILLUSTRATIVE SPECIAL CASE

Equation (13) is valid provided only that the ionosphere exhibits lateral gradients that are much smaller than vertical gradients. In practical terms, this validity condition should be well satisfied for ionospheric disturbances having characteristic lateral dimensions of at least several tens of kilometers. Most types of ionospheric disturbances fall into this category. Once such a disturbance has been defined, and the corresponding values of $S^2(x,y) - S_0^2$ have been found over sufficiently fine grid in the x-y plane, Eq. (13) is readily solved by well-known numerical methods used in the analysis of ground-wave propagation over nonuniform terrain (e.g., *de Jong, 1975*). Such numerical solutions are particularly straightforward at ELF because the exponent in Eq. (13) is so slowly varying.

The goal of this report--a determination of the relative errors incurred by neglecting transverse gradients in ionospheric disturbances--can be achieved without resorting to a full-fledged numerical solution of Eq. (13). Of course, numerical solutions (which will be the subject of a future report) are required to accurately compute the ELF propagation anomaly caused by some specific disturbance. Here, we consider an ionospheric disturbance characterized by

$$S^2(x',y') - S_0^2 = \Delta S^2 \exp\left\{-\frac{\left[x' - x_0\right]^2}{(\Delta x)^2}\right\} \exp\left\{-\frac{\left[y' - y_0\right]^2}{(\Delta y)^2}\right\} . \qquad (14)$$

The analytic form for $S^2 - S_0^2$ given by Eq. (14) is, in fact, a remarkably good fit to ionospheric disturbances caused by high-altitude nuclear bursts (e.g., *Field and Engel, 1965*). Moreover, by adjusting ΔS^2, $(\Delta x)^2$, $(\Delta y)^2$,

x_o, and y_o, Eq. (14) can be used to represent disturbances exhibiting various strengths, longitudinal and transverse gradients, and longitudinal and transverse positions, respectively. Here, we will concentrate on analyzing the effects of nonzero transverse gradients (finite Δy) and off-path (nonzero y_o) disturbances.

Substitution of Eq. (14) into Eq. (13) gives

$$W(x) = 1 + \frac{k^{3/2}\,\Delta S^2\, e^{-\pi i/4}}{2\sqrt{2\pi\, S_o}} \iint dx'dy' \left[\frac{x}{r_1 r_2}\right]^{\frac{1}{2}} \left[\frac{x'}{r_1}\right] W(x',y')$$

$$\exp\left\{-\left[\frac{(x'-x_o)^2}{(\Delta x)^2} + \frac{(y'-y_o)^2}{(\Delta y)^2} + ikS_o(r_1 + r_2 - x)\right]\right\} \quad . \qquad (15)$$

Equation (15) is solved approximately in the appendix, where, subject to the validity criteria

$$kS_o x >> 1 \quad , \qquad (16a)$$

$$\frac{k(S - S_o)(\Delta x)}{2} << 1 \quad , \qquad (16b)$$

and

$$\frac{S - S_o}{4S_o} << 1 \qquad (16c)$$

being satisfied, the following expression is found for the attenuation function:

$$W(x) \approx 1 - \frac{i\sqrt{\pi}}{2}\frac{(\Delta S^2)}{S_o}(k\Delta x)\Gamma^{\frac{1}{2}}(x,x_o,\Delta y)\ \exp\left[-\frac{y_o^2}{(\Delta y)^2}\Gamma(x,x_o,\Delta y)\right] \quad . \qquad (17)$$

In Eq. (17), the quantity

$$\Gamma(x,x_0,\Delta y) = \frac{ikS_0x}{ikS_0x + \frac{2x_0(x - x_0)}{(\Delta y)^2}} \tag{18}$$

has the physical significance of a transverse shape factor that approaches unity as the lateral gradients vanish; i.e., Γ approaches unity as $\Delta y \to \infty$.

An often-used method of treating ELF propagation in a laterally non-uniform earth-ionosphere waveguide is to invoke the so-called "two-dimensional WKB" approximation. In this approximation, longitudinal (x) gradients are accounted for via the usual WKB method, but transverse (y) gradients are neglected to the extent that the characteristics of the disturbance on the direct propagation path (y=0) are assumed to persist for all values of y. This approximation is equivalent to assuming $\Delta y \to \infty$ in Eq. (18); in which case, the transverse shape factor, Γ, is unity, and Eq. (17) becomes

$$W_{WKB} \approx 1 - \frac{i\sqrt{\pi}}{2} \frac{(\Delta S^2)}{S_0} (k\Delta x)\, e^{-y_0^2/(\Delta y)^2} \quad . \tag{19}$$

Equation (19) correctly accounts for the fact that the center of the disturbance is displaced a transverse distance, y_0, from the direct path, but neglects the finite transverse width of the disturbance.

The well-known form of the two-dimensional WKB result is

$$W_{WKB} = \exp\left\{- ik \int_0^x \left[S(x',o) - S_0\right] dx'\right\} \quad , \tag{20}$$

or

$$W_{WKB} \approx 1 - ik \int_0^x \left[S(x';o) - s_o\right]dx' \quad . \tag{21}$$

For the situation of interest here, $0 < x_o < x$, and $S^2 - S_o^2 \approx 2S_o(S - S_o)$. Use of Eq. (14) shows that Eq. (19) and Eq. (21) are identical, provided that the inequality (16b) is satisfied.

To best illustrate the effects of finite lateral gradients, it is convenient to normalize the attenuation function to its value as calculated by neglecting lateral gradients altogether. This value is obtained by setting $\Delta y = \infty$ in either Eq. (17) or Eq. (19), which gives

$$W_{\Delta y=\infty} = 1 - \frac{i\sqrt{\pi}}{2} \frac{(\Delta S^2)}{S_o} (k\Delta x) \quad . \tag{22}$$

Moreover, we are interested in comparing the propagation anomaly calculated by the fullwave and two-dimensional WKB methods. This anomaly is defined as the amount by which W differs from unity.

Accordingly, using Eqs. (17), (19), and (22), we define the following two quantities for use in presentation of numerical results:

$$R = \frac{W - 1}{(W - 1)_{\Delta y=\infty}} = \Gamma^{\frac{1}{2}}(x,x_o,\Delta y) \exp\left\{- \frac{y_o^2}{(\Delta y)^2} \Gamma(x,x_o\Delta y)\right\} \tag{23}$$

and

$$R_{WKB} = \frac{(W - 1)_{WKB}}{(W - 1)_{\Delta y=\infty}} = \exp\left\{-y_o^2/(\Delta y)^2\right\} \quad . \tag{24}$$

The above two quantities have simple physical interpretations. R is the ratio of the propagation anomaly computed via the fullwave method to that computed by totally ignoring lateral gradients. R_{WKB} is the ratio of the anomaly computed via the two-dimensional WKB method to that computed by totally ignoring lateral gradients. Of course, if $y_0 = 0$, then W_{WKB} and $W_{\Delta y = \infty}$ are identical. Thus, for an on-path disturbance, R is the ratio of the anomaly calculated on a fullwave basis to that computed from the WKB formulation.

IV. RESULTS

The numerical results for the normalized attenuation function are calculated from Eqs. (23) and (24), which compare fullwave results with calculations that ignore lateral gradients. Note that Eq. (23) for R is independent of $S - S_0$ and Δx, which define the strength of the disturbance. That surprising behavior occurs because of fortuitous cancellation, and note must be taken of the fact that Eq. (17), which was used in the derivation of Eq. (23), does depend on $S - S_0$ and Δx, and is accurate only if the validity criteria in Eq. (16) are satisfied. Thus, although the results are parameterized only in terms of Δy and y_0, they are valid only if the pathlength, x, exceeds several megameters, the effective longitudinal dimension, Δx, of the disturbance is less than a few megameters, and the "strength," $S - S_0$, of the disturbance is no greater than a few tenths. All results given below are calculated for a pathlength, x, of 6 Mm, and a disturbance centered at the longitudinal position, $x_0 = 3$ Mm.

Figure 1 applies to an on-path (y_0=0) disturbance, and gives the ratio of the propagation anomaly computed by the fullwave method to that computed by the WKB method. The fullwave and WKB methods would be in perfect agreement if $|R|$, as shown in Fig. 1, were unity, and the phase of R were zero. Figure 1 shows that $|R|$ is always less than unity, which shows that the two-dimensional WKB approximation overestimates the anomalous attenuation caused by an on-path disturbance. This behavior occurs because the WKB method characterizes a disturbance centered on the direct propagation path solely by its maximum strength, rather than by some appropriate transverse average.

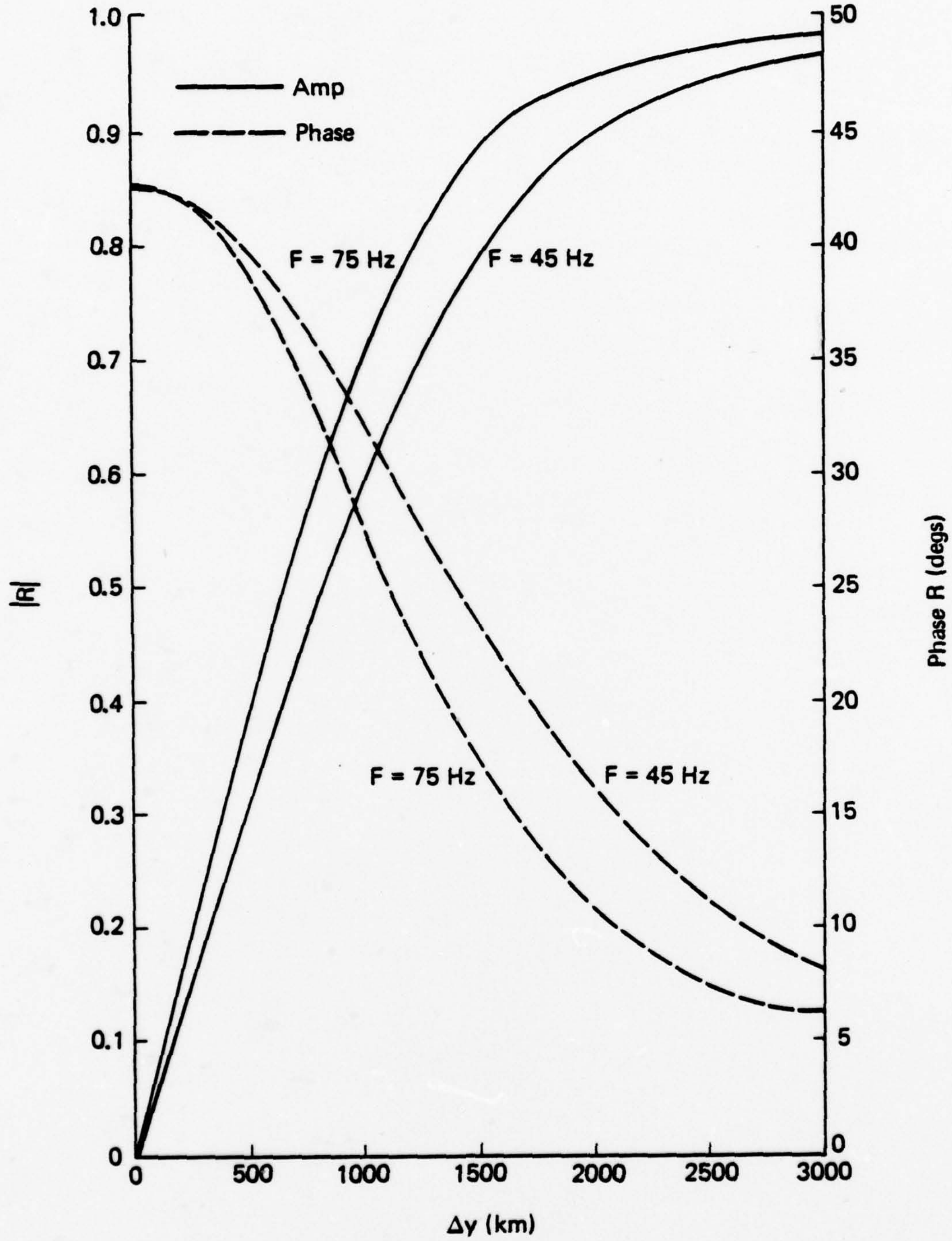

Fig. 1--Ratio of propagation anomaly calculated by the fullwave method to that calculated by the two-dimensional WKB method for an on-path (y_0=0) disturbance; 6-Mm pathlength

Figure 1 also shows that the WKB approximation gives good results (i.e., $|R| \sim 1$) provided that an on-path disturbance has a characteristic transverse dimension, Δy, exceeding about 1500 km. For smaller transverse dimensions, the error incurred by using the WKB approximation and, hence, by neglecting lateral gradients, is seen to be severe. As expected, the difference between the fullwave and WKB methods is larger at 45 Hz than at 75 Hz.

A simple interpretation of the results shown in Fig. 1 can be made by rewriting Eq. (18) for the transverse shape factor in the form

$$\Gamma = \frac{1}{1 - \frac{i}{\pi}\left(\frac{d}{\Delta y}\right)^2} \quad , \tag{25}$$

where d is the half-width of the first Fresnel zone defined by

$$d = \frac{1}{2}\left(\frac{x\lambda}{S_o}\right)^{\frac{1}{2}} \quad , \tag{26}$$

where λ is the free-space wavelength. If the effective half-width, Δy, of the disturbance is greater than the half-width, d, of the first Fresnel zone, Eq. (25) shows that Γ approaches unity, and the WKB results approach the fullwave results. Conversely, if the disturbance is much narrower than a Fresnel zone (i.e., Δy is much smaller than d), then Eq. (18) shows that Γ differs significantly from unity. For this situation, Eq. (23) gives

$$|R| \approx \frac{\sqrt{\pi}(\Delta y)}{d} \quad \text{if} \quad \left(\frac{\Delta y}{d}\right) << 1 \quad . \tag{27}$$

Equation (27) gives the physically reasonable result that, for a localized on-path disturbance, the ratio of the actual attenuation to that calculated from the WKB method is essentially the fraction of the first Fresnel zone that the disturbance fills.

For the parameters used in Fig. 1, the Fresnel-zone half-width, d, is about 2300 km at 75 Hz and 2800 km at 45 Hz. The figure thus shows that the two-dimensional WKB method gives reasonably accurate results provided that the effective transverse width of the disturbance exceeds about two-thirds of the first Fresnel zone. For more localized on-path disturbances, the WKB approximation seriously overestimates the magnitude of the propagation anomaly.

Figures 2 and 3 show the effect of moving the center of a localized disturbance progressively farther off-path; i.e., the effects of increasing y_0 are shown for several assumed half-widths, Δy. Figure 2, which applies to a disturbance that is much narrower (Δy = 500 km) than a Fresnel zone, shows the considerable disagreement between the fullwave and WKB that occurs in this situation. Figure 3 applies to disturbances exhibiting much gentler transverse gradients (Δy = 1000 km and 2000 km) than Fig. 1, and shows that the WKB method may be used--and lateral gradients safely ignored--if the effective width of the disturbance approaches 2000 km, which is a significant fraction of the Fresnel-zone-width.

Figures 2 and 3 show that the WKB approach overestimates the effects of disturbances that are nearly centered on the direct path, but underestimates the effects of disturbances that are centered significantly off-path. The reason, of course, is that the WKB method accounts only for the structure of the ionosphere along the direct transmission path. Thus, the region of greatest strength of an on-path disturbance is overemphasized, whereas this region is not accounted for at all for an off-path disturbance.

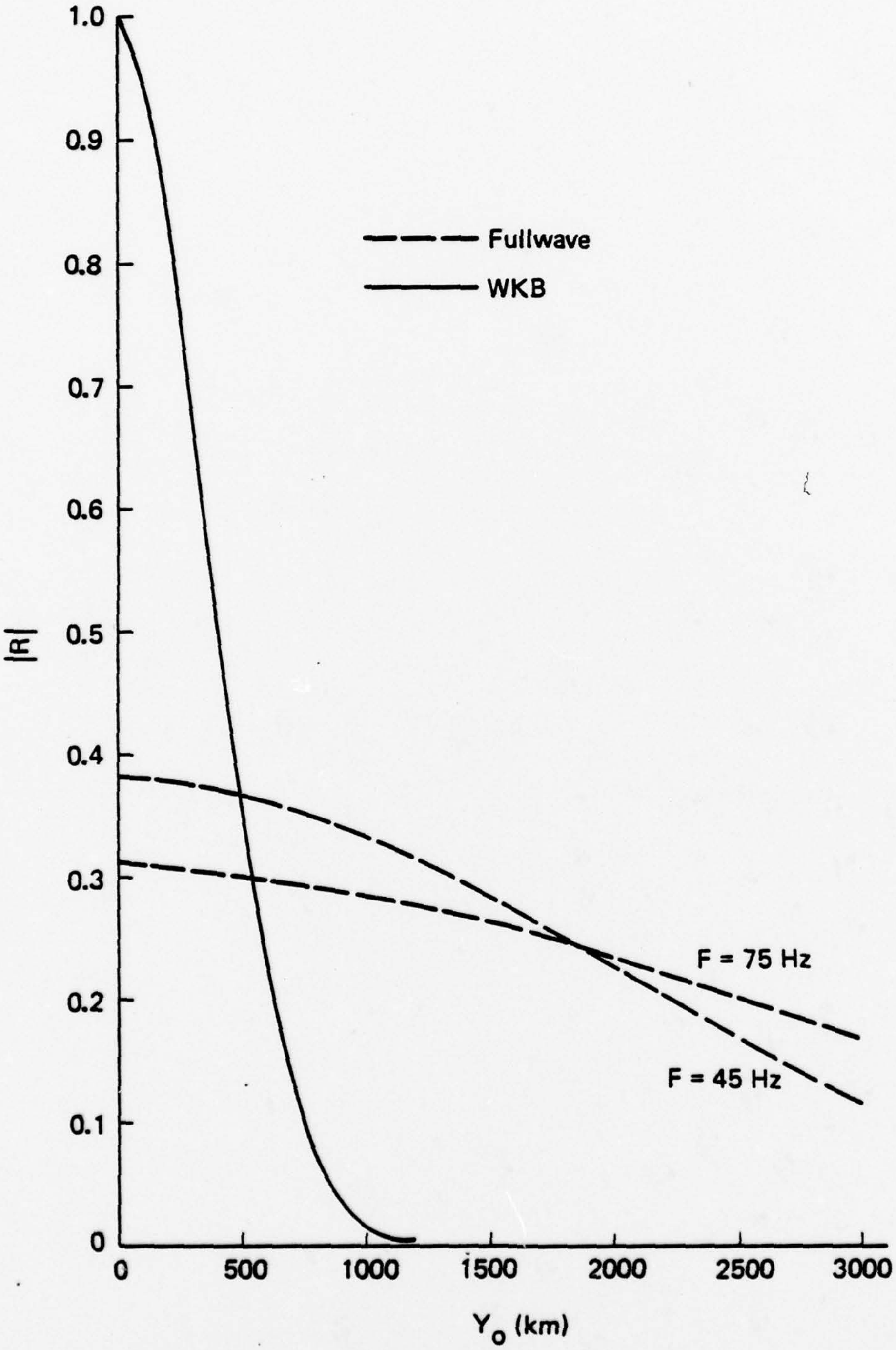

Fig. 2--Comparison of fullwave and WKB calculations for off-path disturbance with effective half-width, Δy, of 500 km; 6-Mm pathlength

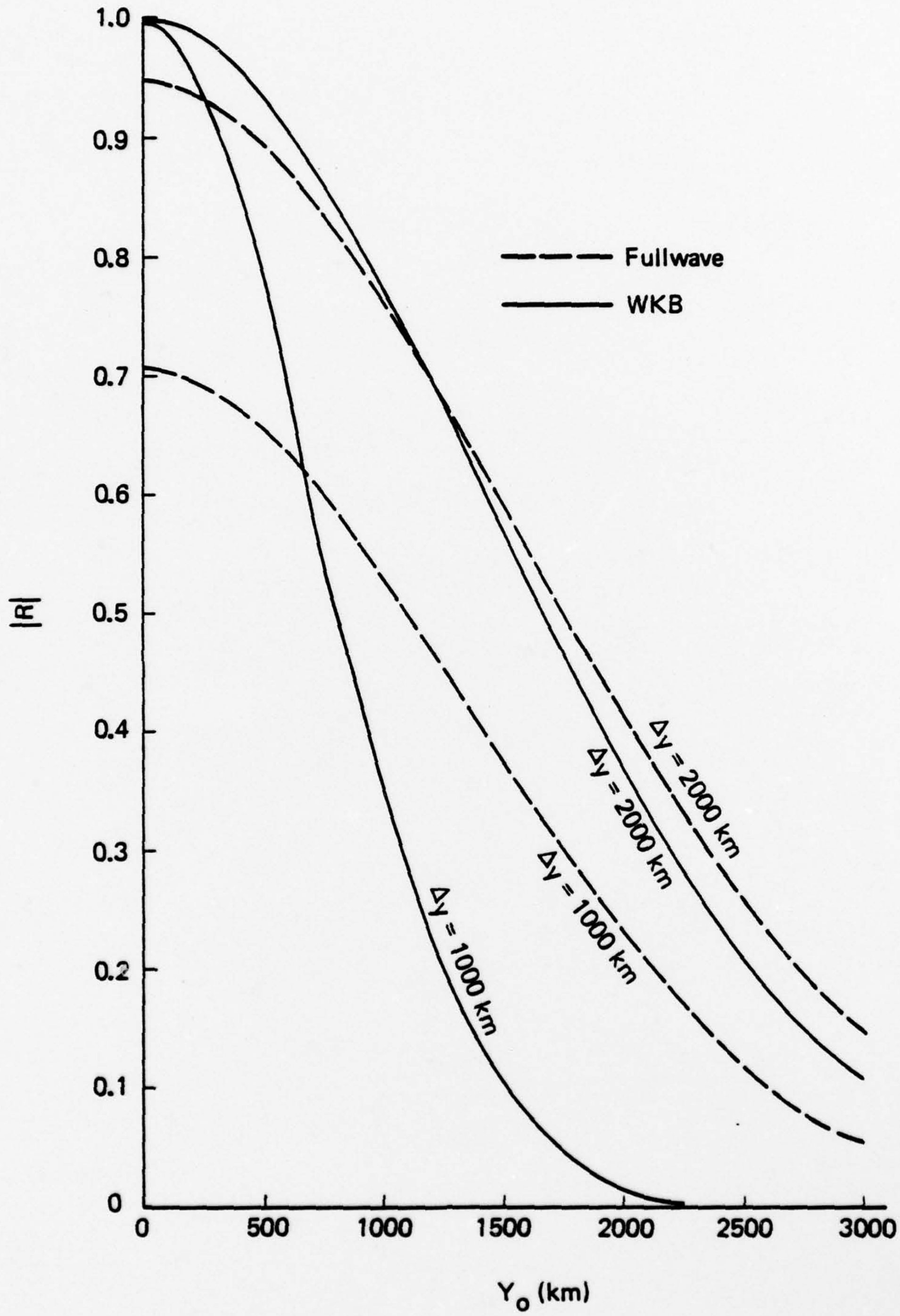

Fig. 3--Comparison of fullwave and WKB calculations for off-path disturbances with effective half-widths of 1000 km and 2000 km; F=75 Hz, 6-Mm pathlength

V. CONCLUSIONS

The above results show that transverse gradients may be ignored--and the two-dimensional WKB method used--if the effective width of an ionospheric depression exceeds about two-thirds of the width of the first Fresnel zone. For typical ELF frequencies and pathlengths, ionospheric depressions with half-widths exceeding 1.5 Mm to 2 Mm satisfy the above criterion. Full-wave ELF calculations must be used to analyze the propagation effects of an ionospheric disturbance that varies significantly over transverse distances less than about 1500 km. For this situation, the WKB approximation seriously overestimates the corresponding propagation anomaly if the disturbance is centered near the direct propagation path, and underestimates the anomaly if the disturbance is centered a significant distance off-path. These conclusions apply to simple ionospheric disturbances that exhibit only a single spatial maximum in the x-y plane. The disturbance, however, need not be cylindrically symmetric. The above conclusions do not apply to complex, multipeaked disturbances, such as would be produced by a number of spatially separated sources.

In the context of nuclear weapons effects, the results given in this report are applicable to single-burst environments. The lateral extent of ionospheric disturbances caused by bursts in the atmosphere depends mainly on the height-of-burst, with yield and time-after-burst also being significant factors. It can be inferred from the work of Knapp and Schwartz (*1975*) and Field and Engel (*1965*) that the effective half-width of prompt disturbances caused by daytime bursts is less than 1 Mm for burst-heights below 100 km, and is less than 500 km for burst-heights below 60 km. Thus, use of fullwave methods that account for transverse ionospheric gradients would appear necessary for burst-heights below about 100 km.

APPENDIX

This appendix derives Eq. (17) from Eq. (15) (see p. 9), and establishes the accuracy of the approximations used. To avoid obscuring the basic principles with algebraic complexity, the derivation is shown for the special case of an on-path disturbance, for which $y_0 = 0$. Although much more complicated algebraically, the derivation when $y_0 \neq 0$ proceeds in essentially the same manner as shown below.

For $y_0 = 0$, most of the contribution to the y'-integration in Eq. (15) comes from the region near $y' = 0$. This behavior occurs because a) the function $\exp[-(y')^2/(\Delta y)^2]$ has a maximum at $y' = 0$, and b) the quantity $r_1 + r_2 - x \approx 0$ for $y' \approx 0$, causing the integrand to be least oscillatory in this region. Thus motivated, we write

$$r_1 \approx x' + (y')^2/2x' + \ldots \tag{A-1}$$

$$r_2 \approx x - x' + \frac{(y')^2}{2(x - x')} + \ldots \tag{A-2}$$

and

$$W(x',y') \approx W(x',o) + \frac{\partial W(x',o)}{\partial y'} (y') + \frac{\partial^2 W(x',o)}{\partial^2 y'} \frac{(y')^2}{2} + \ldots \quad . \tag{A-3}$$

Substitution of Eqs. (A-1) to (A-3) into Eq. (15) leads, after some rearrangement, to the following relation:

$$W(x) \approx 1 + \frac{k^{3/2} e^{-\pi i/4} (\Delta S^2)}{2\sqrt{2\pi S_0}} \int_{-\infty}^{\infty} dx' \; W(x',o) \exp\left[- \frac{(x' - x_0)^2}{(\Delta x)^2} \right] I(x') \quad , \tag{A-4}$$

where

$$I(x') \approx \left[\frac{x}{x'(x-x')}\right]^{\frac{1}{2}} \int_{-\infty}^{\infty} dy' \exp\left[-\left(\frac{1}{(\Delta y)^2} + \frac{ikS_o}{2}\left\{\frac{1}{x'} + \frac{1}{x-x'}\right\}\right)(y')^2\right] \cdot$$

$$\left[1 + \left(\frac{W_{yy}(x',o)}{2W(x',o)} - \left\{\frac{1}{(x')^2} + \frac{1}{2(x-x')^2}\right\}\right)(y')^2 + \ldots\right] \quad . \qquad \text{(A-5)}$$

In Eq. (A-5), the subscript y denotes differentiation with respect to y', and the term involving W_y in Eq. (A-3) vanishes due to the symmetry of the integrand. The integration in Eq. (A-5) can be carried out immediately to give the following result:

$$I(x') \approx \left[\frac{2\pi x}{\frac{2x'(x-x')}{(\Delta y)^2} + ikS_o x'}\right]^{\frac{1}{2}} \cdot \left[1 + \frac{(\Delta y)^2\left[\frac{W_{yy}}{2W} - \frac{1}{(x')^2} - \frac{1}{2(x-x')^2}\right]}{2\left[1 + \frac{ikS_o x(\Delta y)^2}{2x'(x-x')}\right]} + \ldots\right] \quad . \qquad \text{(A-6)}$$

To estimate the error term--the second term in the second brackets--in Eq. (A-6), we consider disturbances that are not too near the transmitter or receiver, so that $x' \sim x/2$. Also, to estimate the order of magnitude of the term $W_{yy}/2W$, we use the approximate expression

$$W \sim \exp\left[-ik(S(x',y') - S_o)\left[(x')^2 + (y')^2\right]^{\frac{1}{2}}\right] \quad , \qquad \text{(A-7)}$$

from which it follows that

$$\frac{W_{yy}(x',o)}{2W(x',o)} \sim -\frac{ik(S(x',o) - S_o)}{2x'} \quad . \qquad \text{(A-8)}$$

Substitution into Eq. (A-6) gives the following expression for the order of magnitude of the error term, ε_1, in Eq. (A-6):

$$\varepsilon_1 \sim \frac{(\Delta y)^2/x^2}{2\left[1+2ikS_o x\left(\frac{\Delta y}{x}\right)^2\right]}\left[ik\Big(S(x/2,0)-S_o\Big)x+6\right] \quad . \tag{A-9}$$

The error term, Eq. (A-9), is largest if

$$\left(\frac{\Delta y}{x}\right)^2 >> 1/2kS_o x \quad ; \tag{A-10}$$

in which case, Eq. (A-9) becomes

$$\varepsilon_1 \sim \frac{(S-S_o)}{4S_o} + \frac{3}{2kS_o x} \quad . \tag{A-11}$$

It is easy to show that if $\Delta y/x$ is smaller than indicated by Eq. (A-10), then the error term is smaller than given by Eq. (A-11). Thus, very conservatively, we may use

$$I(x') \approx \left[\frac{2\pi x}{\frac{2x'(x-x')}{(\Delta y)^2} + ikS_o x}\right]^{1/2} \quad , \tag{A-12}$$

provided that

$$\frac{S-S_o}{4S_o} << 1 \quad , \tag{A-13}$$

and

$$kS_o x >> 1 \quad . \tag{A-14}$$

The inequality (A-13) is usually well satisfied because S_o is of order unity; whereas, $S - S_o$ is typically no greater than a few tenths. Although less valid than at higher frequencies, the inequality (A-14) is fairly well-satisfied at ELF and--in any event--is widely used (e.g., going from Eq. (12) to Eq. (13)) for ELF propagation over pathlengths exceeding a few megameters.

Insertion of Eq. (A-12) into Eq. (A-4) gives the following approximate one-dimensional integral equation:

$$W(x,o) \approx 1 + \frac{k(\Delta S^2)}{2S_o} e^{-\pi i/4} \sqrt{kS_o x} \int_{-\infty}^{\infty} dx' \frac{W(x',o) \exp\left[-\frac{(x'-x_o)^2}{(\Delta x)^2}\right]}{\left[\frac{2x'(x-x')}{(\Delta y)^2} + ikS_o x\right]^{\frac{1}{2}}} . \quad \text{(A-15)}$$

By expanding $W(x',o)$ and the denominator of the integrand in Eq. (A-15) about $x' = x_o$, a series representation of the integral is obtained. The steps are essentially the same as described above for the y'-integration; and the result is

$$W(x,o) \approx 1 - i \frac{\sqrt{\pi}}{2} \frac{(\Delta S^2)}{S_o} (k\Delta x) \; \Gamma^{\frac{1}{2}} (x,x_o,\Delta y) \; W(x_o,o) \left[1 + \varepsilon_2\right] \quad , \quad \text{(A-16)}$$

where Γ is defined by Eq. (18), and the error term, ε_2, is

$$\varepsilon_2 \sim \left[\frac{W_{xx}}{2W}\bigg|_{x=x_o} + \frac{2}{2x_o(x-x_o) + ikS_o x(\Delta y)^2}\right] \frac{[\Delta x]^2}{2} \quad \text{(A-17)}$$

for the situation where $x_o \sim x/2$. The subscript on W denotes x'-differentiation. By using

$$W(x,o) \sim \exp\left[- ik \int_{o}^{x} \left(S(x',o) - S_o\right) dx' \right] \tag{A-18}$$

to estimate the order of magnitude of W_{xx}, it follows that

$$\varepsilon_2 \sim \left(\frac{k\Delta x\left(S(x_o,o) - S_o\right)}{2}\right)^2 + \frac{(\Delta x)^2}{2x_o(x - x_o) + ikS_o x(\Delta y)^2} \quad . \tag{A-19}$$

Provided that Δx is not much larger than Δy, the second term in Eq. (A-19) is small if the previously established condition, Eq. (A-14), is satisfied. Thus, the only additional constraint imposed by the approximate x'-integration is that

$$\left(\frac{k\Delta x(S - S_o)}{2}\right)^2 < < 1 \quad . \tag{A-20}$$

For typical ELF paths, $k \sim 10^{-3}$ km^{-1} and $(S - S_o)/2 \sim 0.1$. Thus, the condition (A-20) will be satisfied if $\Delta x \lesssim 3$ Mm or so.

Subject to the above constraints, the error term in Eq. (A-16) may be neglected, and the solution is easily shown to be

$$W(x) \approx 1 - \frac{\frac{\sqrt{\pi i}}{2} \frac{(\Delta S^2)}{S_o} (k\Delta x) \quad \Gamma^{\frac{1}{2}}(x,x_o,\Delta y)}{1 + i \frac{\sqrt{\pi}}{2} \frac{(\Delta S^2)}{S_o} k(\Delta x)} \quad . \tag{A-21}$$

For off-path disturbances, where $y_o \neq 0$, the algebra involved in approximately solving Eq. (15) is more complicated than given above for the on-path disturbance. Moreover, if the disturbance is centered very

far off-path, most of the contribution to the y'-integration in Eq. (15) no longer is strongly peaked about $y' = 0$. The main consequence of this fact is that the one-dimensional equation that results after performing the approximate y'-integration involves W at nonzero values of y. This equation cannot be solved in the manner of Eqs. (A-15) and (A-16), which involved only W on the line $y = 0$. To deal with this complexity, it is necessary to assume $W = 1$ on the right-hand side of the integral equation, which is equivalent to using first-order perturbation theory.

Thus, for $y_0 \neq 0$, we use $W = 1$ in the right-hand side of Eq. (A-4), and proceed in the manner used in going from Eq. (A-4) to Eq. (A-12). The new form of the function, I, is (neglecting correction terms)

$$I(x',y_0) \approx \left[\frac{x}{x'(x-x')}\right]^{1/2} \int_{-\infty}^{\infty} dy' \, \exp\left[-\left(\frac{ikS_0}{2}\left\{\frac{1}{x'} + \frac{1}{x-x'}\right\} + \frac{1}{(\Delta y)^2}\right)(y')^2 + \frac{2y_0 y'}{(\Delta y)^2}\right]$$

$$= \frac{\sqrt{2\pi x}}{\left[ikS_0 x + \frac{2x'(x-x')}{(\Delta y)^2}\right]^{1/2}} \exp\left[-\frac{y_0^{\,2}}{(\Delta y)^2}\,\Gamma(x,x',\Delta y)\right] \quad . \qquad \text{(A-22)}$$

By performing the x'-integration as outlined above for the on-path disturbance, it is easy to obtain Eq. (17), which is accurate provided that the first-order validity criterion,

$$\frac{k(S - S_0)\,(\Delta x)}{2} << 1 \quad , \qquad \text{(A-23)}$$

is satisfied. The criterion (A-23), which applies for off-path disturbances, is somewhat more stringent than the second-order validity criterion (A-20), which applies for on-path disturbances (see Eq. (A-21)). Note that for $y_0 = 0$,

Eq. (17) differs from Eq. (A-21) by the term

$$1 + \frac{i\sqrt{\pi}}{2} \frac{\Delta S^2}{S_0} \; k(\Delta x)$$

that appears in the denominator on the right-hand side of Eq. (A-21). This difference is due solely to the fact that Eq. (A-21) is accurate to second order in $k(S - S_0)(\Delta x)$, whereas Eq. (17) is accurate only to first order. This slight inconsistency between the expressions for the on-path and off-path disturbances vanishes due to cancellation in the function $R = (W-1)/(W-1)_{\Delta y=\infty}$ (see Eq. (23)), which is independent of ΔS^2 and x, and is used to present the numerical results.

REFERENCES

Budden, K. G., *The Waveguide Mode Theory of Wave Propagation,* Logos Press, London, 1961.

de Jong, G., "Electromagnetic wave propagation over an inhomogeneous flat earth" (two-dimensional integral equation formulation), *Radio Science,* Vol. 10, November 1975, pp. 925-933.

Field, E. C., "The Effects of Ions on Very-Low-Frequency Propagation During Polar Cap Absorption Events," *Radio Science,* Vol. 5, March 1970, p. 591.

Field, E. C., and R. D. Engel, "The Detection of Daytime Nuclear Bursts Below 150 km by Prompt VLF Phase Anomalies," *Proc. IEEE,* Vol. 53, December 1965, p. 2009.

Galejs, J., *Terrestrial Propagation of Long Electromagnetic Waves,* Pergamon Press, New York, 1972.

Greifinger, C., and P. Greifinger, *Effect of a Cylindrically-Symmetric Ionospheric Disturbance on ELF Propagation in the Earth-Ionosphere Waveguide,* R&D Associates, Marina del Rey, California, DNA 43394, 10 June 1977.

Hufford, G. A., "An Integral Equation Approach to the Problem of Wave Propagation Over an Irregular Surface," *Quart. Appl. Math, 9,* 1952, pp. 391-403.

Knapp, W. S., and K. Schwartz, *Aids for the Study of Electromagnetic Blackout,* General Electric Company (TEMPO), DNA 3499H, 1 February 1975.

Pappert, R. A., and W. F. Moler, "Propagation Theory and Calculations at Lower Extremely Low Frequencies (ELF)," *IEEE Trans. on Communications,* Vol. COM-22, April 1974, pp. 438-451.

Wait, J. R., "On Phase Changes in Very-Low-Frequency Propagation Induced by an Ionospheric Depression of Finite Extent," *Journ. Geophys. Res.,* Vol. 59, No. 3, 1 Febraury 1964, pp. 441-445.

-----, *Electromagnetic Waves in Stratified Media,* Pergamon Press, New York, 1970.

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